North Sydney Girls High School

2022 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 9 – 44)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

NAME:_____ **TEACHER:**_____

STUDENT NUMBER:

Question	1–10	11–22	23–29	30–35	Total
Mark	/10	/33	/30	/27	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 Which of the following is equivalent to $\sqrt{3}+\sqrt{27}-\sqrt{18}$?

A. $\sqrt{3}$

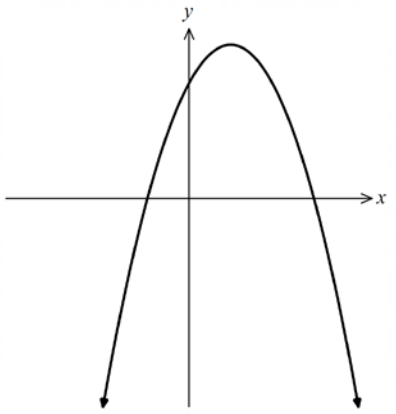
B. $\sqrt{12}$

C. $4\sqrt{3}-3\sqrt{2}$

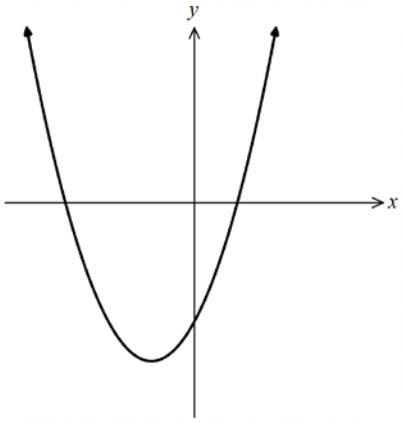
D. $10\sqrt{3}-9\sqrt{2}$

2 Which of the following graphs best represents $y=x^2+2x-3$?

A.



B.



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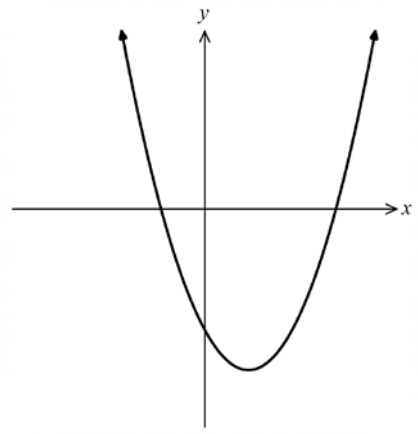

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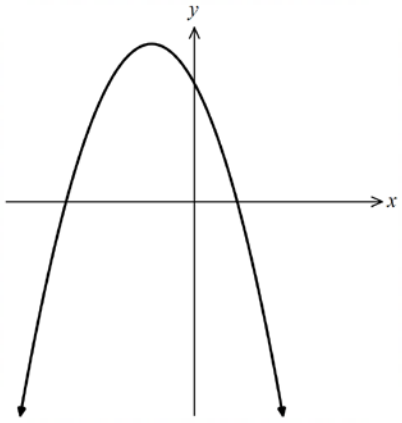


3 Two standard unbiased dice (with faces numbered 1 to 6) are rolled.
What is the probability of rolling at least one six?

A. $\frac{1}{6}$

B. $\frac{11}{36}$

C. $\frac{25}{36}$

D. $\frac{5}{6}$

4 What is the solution to the inequation $x^2 + 4x + 3 \geq 0$?

A. $x \leq -1$ or $x \leq -3$

B. $x \leq -1$ or $x \geq -3$

C. $x \geq -1$ or $x \geq -3$

D. $x \geq -1$ or $x \leq -3$

5 Which of the following is $\frac{dy}{dx}$ if $y = 2\tan(3-2x)$?

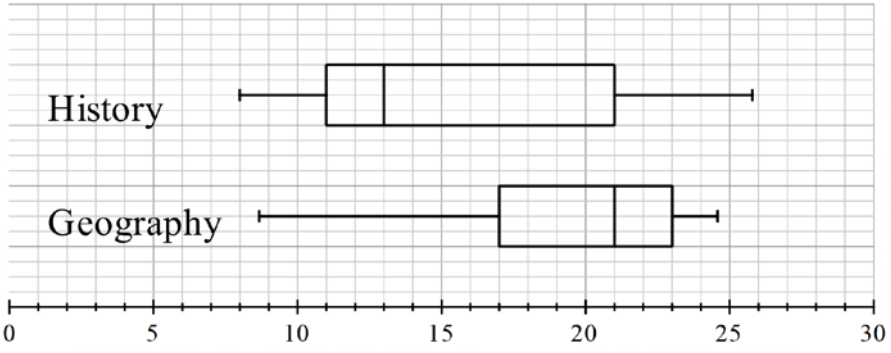
A. $\frac{dy}{dx} = 2\sec^2(3-2x)$

B. $\frac{dy}{dx} = 6\sec^2(3-2x)$

C. $\frac{dy}{dx} = -2\sec^2(3-2x)$

D. $\frac{dy}{dx} = -4\sec^2(3-2x)$

6 The box-and-whisker plots below show the results of a History and Geography test.



In History 116 students completed the test. The number of students who scored above 21 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 21

B. 42

C. 58

D. 116

7 For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > -\frac{1}{6}$

C. $x < -6$

D. $x > 6$

8 A function is defined by $y = \sin^2 \sqrt{x-2}$.
Which of the following gives the domain and range of this function?

A. Domain: $x \geq 0$ and Range: $-1 \leq y \leq 1$

B. Domain: $x \geq 2$ and Range: $-1 \leq y \leq 1$

C. Domain: $x \geq 0$ and Range: $0 \leq y \leq 1$

D. Domain: $x \geq 2$ and Range: $0 \leq y \leq 1$

9 The only stationary point on the graph $y = f(x)$ is the point (a,b).
What are the coordinates of the only stationary point on the graph $y = -f(2x)$?

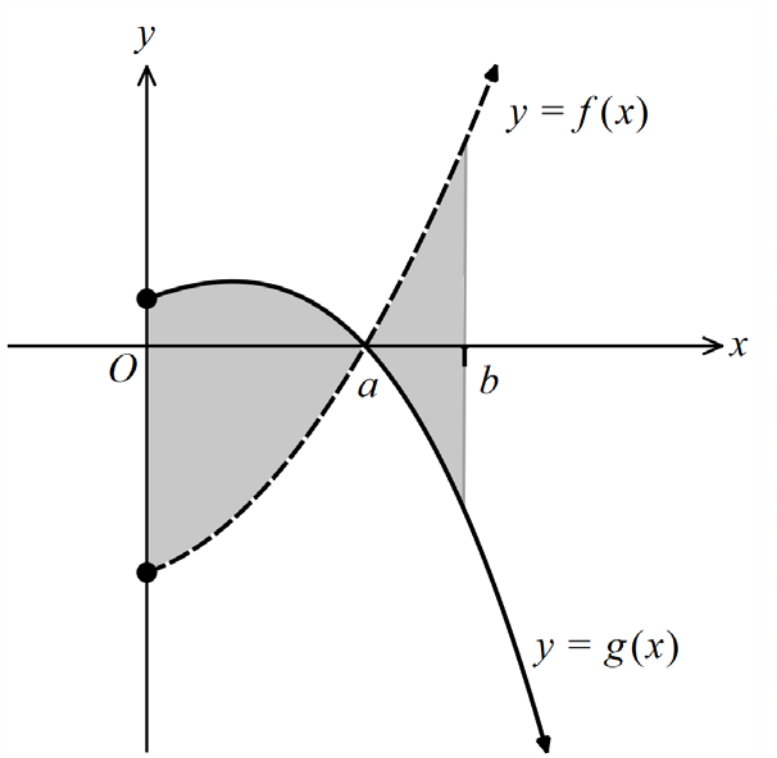
A. $\left(\frac{1}{2}a, -b\right)$

B. $(2a, -b)$

C. $\left(-\frac{1}{2}a, b\right)$

D. $(-2a, b)$

10 The functions $f(x)$ and $g(x)$ are shown in the diagram below.



Which of the following is NOT a correct expression for the shaded area?

A. $A = \int_0^b \left| f(x) - g(x) \right| dx$

B. $A = \int_0^b \left(\left| f(x) \right| - \left| g(x) \right| \right) dx$

C. $A = \int_0^b \left| f(x) \right| dx + \int_0^b \left| g(x) \right| dx$

D. $A = \int_0^a \left(g(x) - f(x) \right) dx + \int_a^b \left(f(x) - g(x) \right) dx$

Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–35
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–22 (33 marks)
Booklet 2 – Attempt Questions 23–29 (30 marks)
Booklet 3 – Attempt Questions 30–35 (27 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 11 (2 marks)

100 users of a social media app were surveyed and their ages are shown in the following table.

Age	$15 \le x < 25$	$25 \le x < 35$	$35 \le x < 45$	$45 \le x < 55$	$55 \le x < 65$
Number of app users	41	30	15	8	6
Class Centre	20	30	40	50	60

Calculate the mean and standard deviation for this data, correct to two decimal places. **2**

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Question 12 (2 marks)

Evaluate $2\int_0^1 7^{-2x}\,dx$ correct to three decimal places. **2**

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Question 13 (3 marks)

Find $\frac{d}{dx}\left(\frac{e^{2x}}{\sin(x)}\right)$. **3**

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Question 14 (2 marks)

The fourth term of a geometric series with common ratio $r = -0.9$ is -21141.

(a) Find the first term. **1**

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(b) Find the sum of the first 18 terms, correct to one decimal place. **1**

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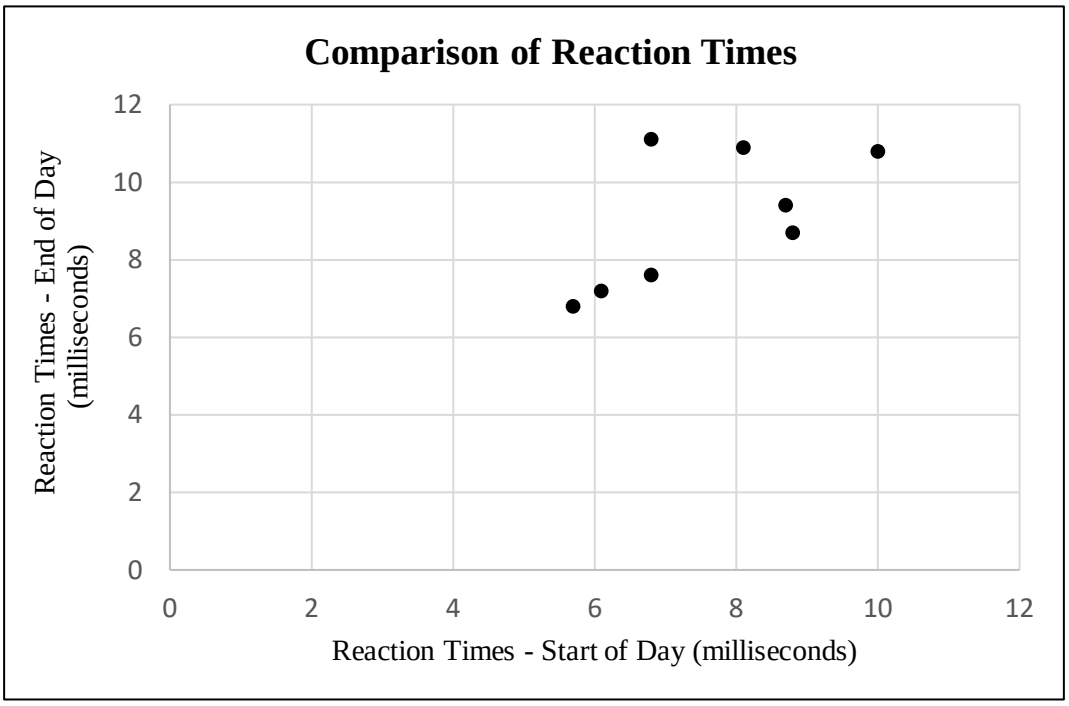
Question 15 (6 marks**)**

A random sample of eight students is selected from a school database. Each student's reaction time is measured at the start and end of the school day. The reaction times in milliseconds are shown in the table and scatterplot below.

Student	A	B	C	D	E	F	G	H
Reaction time: Start of day	10.0	6.1	8.8	5.7	8.7	8.1	6.8	6.8
Reaction time: End of day	10.8	7.2	8.7	6.8	9.4	10.9	11.1	7.6



(a) Sketch a line of best fit on the graph above. **1**

(b) Find the correlation coefficient and describe the association in this data. **2**

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Question 15 continues on Page 13

(c) Find the equation of the least squares regression line. Give coefficients to two decimal places. **1**

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(d) Use your equation in (c) to predict the reaction time at the end of the day for a student who had a reaction time of 6.8 seconds at the start of the day. **2**
Compare this value with the data for the relevant students from the table.

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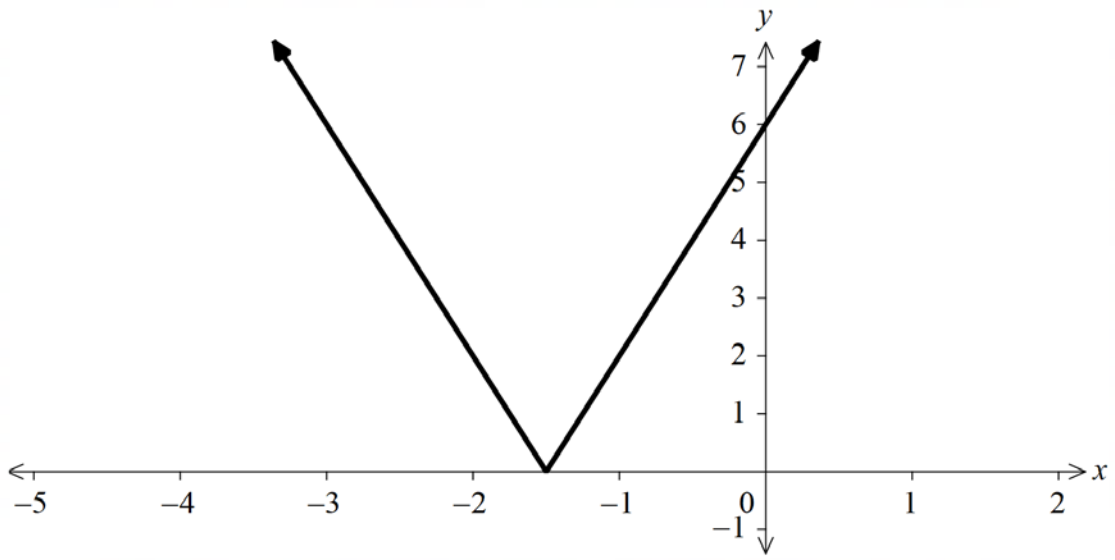
End of Question 15

Question 16 (2 marks)

The graph of $f(x) = |ax + b|$ is shown below. **2**



Find the value of *a* and *b*.

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Question 17 (3 marks)

Solve $\frac{\cos^3\theta}{\sin\theta} + \sin\theta\cos\theta = 1$ in the domain $0 \le \theta \le 2\pi$. **3**

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Question 18 (4 marks**)**

The temperature T inside an oven, measured in degrees Celsius, m minutes after it is switched on, can be modelled by the equation $T = 200 - 175e^{-km}$, where k is a constant.

(a) Write down the initial temperature of the oven. **1**

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(b) If the oven reached a temperature of 150°C after 16 minutes and 15 seconds, find the value of k correct to three significant figures. **2**

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(c) What will be the temperature in the oven after 10 minutes? **1**

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Question 19 (2 marks)

A shoe store buys shoes for \$50 a pair and sells them for \$80. If their weekly rent and salary costs are \$2100 a week, how many pairs must they sell each week to break even? **2**

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Question 20 (2 marks)

Find $\int x\sin(x^2)dx$. **2**

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Question 21 (2 marks)

Evaluate 10^x, given that $\frac{4^x}{2^{x+1}} = \frac{3}{5^x}$. (You are not required to find the value of x.) **2**

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Question 22 (3 marks)

(a) Sketch the graph of $y = 3\sin\left(\frac{x}{2}\right)$, for the interval $-2\pi \le x \le 2\pi$. **2**

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(b) Hence or otherwise find the number of solutions to the equation $3\sin\left(\frac{x}{2}\right) = x + 1$ in the domain $-2\pi \le x \le 2\pi$. **1**

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Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 23–29 (30 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 23 (4 marks**)**

Given that $f(x) = x^2 + 3$ and $g(x) = x + 4$:

(a) Find simplified expressions for $f(g(x))$ and $g(f(x))$. **2**

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(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **2**

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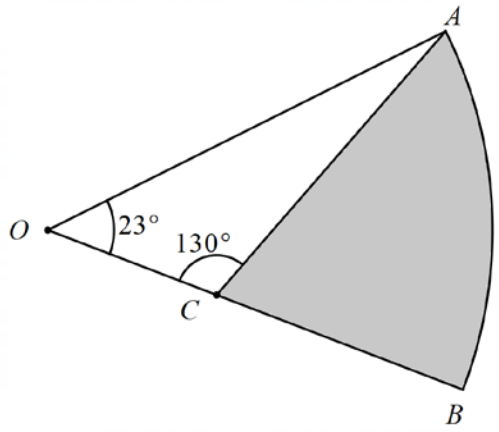
Question 24 (5 marks)

The diagram shows *OAB*, the sector of a circle centred at *O*. Angle $\angle AOB$ is 23° and angle $\angle ACO$ is 130°. The line segment *AC* divides the sector and is 7 cm long.



NOT TO SCALE

(a) Find the length of *OA*, the radius of the circle. **2**

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(b) Find the area of the shaded region. **3**

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Question 25 (2 marks**)**

Differentiate $f(x) = 3 - x^2$ from first principles. **2**

You may use $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

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Question 26 (3 marks**)**

A discrete random variable X has the following probability distribution.

x	2	4	6	10
$P(X = x)$	0.3	b	0.1	c

It is known that the expected value, $E(X)$, is 4.8.

(a) Show that the values of b and c are 0.4 and 0.2 respectively. **2**

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(b) Calculate the variance for this probability distribution. **1**

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Question 27 (7 marks**)**

Consider the curve $y = 3x^4 - 4x^3 + 2$.

(a) Find the coordinates of any stationary points on the curve and determine their nature. **3**

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(b) Find the coordinates of any points of inflection on the curve. **2**

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Question 27 continues on Page 27

(c) Hence, or otherwise, sketch the graph of $y = 3(x-1)^4 - 4(x-1)^3 + 2$, showing the coordinates of the stationary points and points of inflection. **2**

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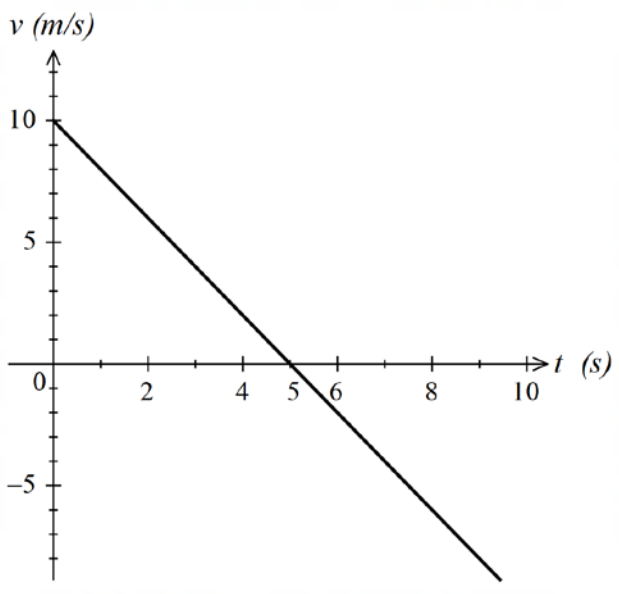
End of Question 27

Question 28 (6 marks)

The velocity-time graph of a particle moving in a straight line is shown below.



(a) Describe the motion of the particle in the first 8 seconds of the motion. **2**

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(b) Find the acceleration of the particle. **1**

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Question 28 continues on Page 29

(c) Find the total distance travelled by the particle in the first 8 seconds. **2**

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(d) If initially the particle is two units to the left of the origin, what is the location of the particle after 8 seconds? **1**

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Question 29 (3 marks**)**

Solve the equation $\tan^2\left(x-\frac{\pi}{6}\right)+\left(\sqrt{3}+1\right)\tan\left(x-\frac{\pi}{6}\right)=-\sqrt{3}$, where $0\le x\le\pi$. **3**

Give answers in exact form.

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Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 30–35 (27 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 30 (4 marks**)**

Brendan and Michelle are taking turns in an eating competition. **4**
In the first round, Brendan eats one M&M, with a volume of 0.5 cm^3, or half a millilitre, and Michelle then follows by eating two M&Ms.
In the second round, Brendan eats three M&Ms and then Michelle eats four.
If they continue in this manner, how many M&Ms will Brendan eat in the final round before he runs out of room in his stomach, if it can hold 3 Litres?

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Question 31 (5 marks)

A bag contains 2 white and 5 yellow tokens.
In a group of people, each person draws a token randomly from the bag, with replacement.

If the token is white, they answer the question "Are you a smoker?"
If a yellow token is drawn, they answer the question "Are you a non-smoker?"

They then answer either "Yes" or "No", depending on whether they smoke or not.

Let p be the probability that a randomly selected person is a smoker.

(a) Show that the probability that a randomly selected person answered "Yes" is given by: **1**

$$P(Yes) = \frac{5-3p}{7}.$$

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(b) It is now known that the group has 91 people and 50 of them answered "Yes".

(i) Find p. **2**

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(ii) Given a person answered "No", find the probability that they are a non-smoker. **2**

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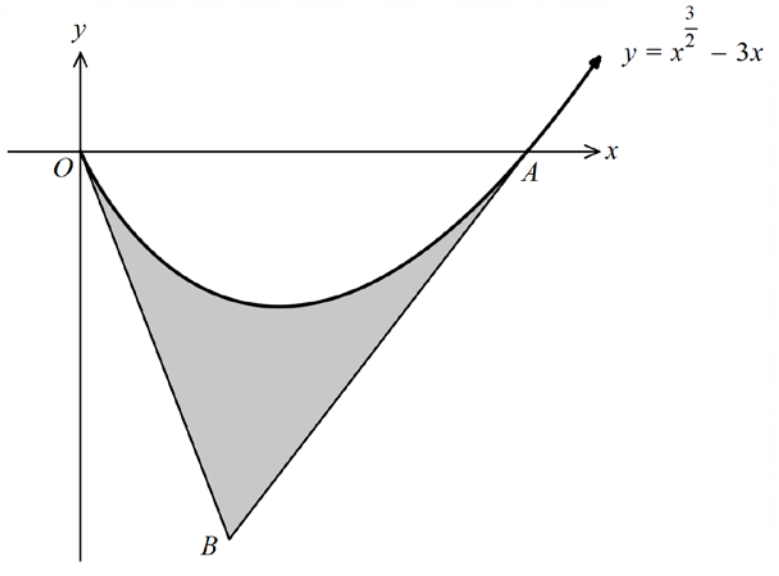
Question 32 (6 marks)

The diagram shows the graph of $y = x^{\frac{3}{2}} - 3x$, which meets the x-axis at the origin and at the point $A(9, 0)$. Tangents drawn to the curve at O and A meet at the point B.



(a) Show the coordinates of B are $(3, -9)$. **3**

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Question 32 continues on Page 37

Question 32 (continued)

(b) Hence find the area of the shaded region bounded by the two lines and the curve. **3**

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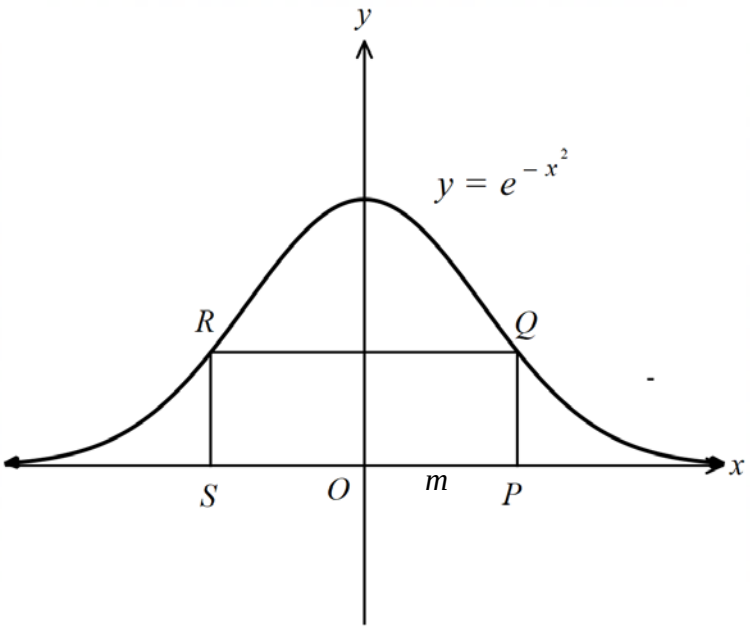
End of Question 32

Question 33 (4 marks)

The diagram shows a rectangle *PQRS*, where *Q* and *R* are points on the curve $y = e^{-x^2}$ and *P* and *S* are on the *x*-axis. *O* is the origin and $OP = OS$. **4**



Let the length $OP = m$. Find the value of *m* that maximises the area of *PQRS*.

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Question 34 (2 marks**)**

Given that $-\frac{\pi}{2} < x < \frac{\pi}{2}$, simplify the expression below as a single trigonometric ratio, **2**

$$\sqrt{1+\sin^2 x+\sin^4 x+\sin^6 x+\ldots}$$

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Question 35 (6 marks)

An engineer models the rates of change of the amount of oil produced (in hundred barrels per day) by oil companies *A* and *B* respectively by

$$f'(t) = 6\ln(t) \quad \text{and} \quad g'(t) = \frac{60t}{1+5t} \ ,$$

where *t* is the number of days after some initial time $(2 \le t \le 14)$.

(a) Using the trapezoidal rule with 3 sub-intervals, estimate the total amount of oil produced by oil company *A* from $t = 2$ to $t = 14$. **2**

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(b) It can be shown that $\frac{5t}{1+5t} = 1 - \frac{1}{1+5t}$. DO NOT prove this. **1**

Use this result to find $\int \frac{60t}{1+5t}\,dt$.

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Question 35 continues on Page 41

(c) The engineer claims that the total amount of oil produced by oil company *A* from $t = 2$ to $t = 14$, is less than that of oil company *B*. Do you agree? Justify your answer. **3**

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End of paper

North Sydney Girls High School

2022 **HSC TRIAL EXAMINATION**

Mathematics Advanced

SOLUTIONS AND FEEDBACK

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- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
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Total marks: 100

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- Attempt Questions 1 – 10
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Markers Comments

Some general comments:

DO write only in the space provided. Working for any question should only be in the writing space provided – not in the empty space alongside the graph or diagram, not at the top or bottom of the page or the margins. Especially not the margins as the booklets are chopped and scanned and some part of your response may be lost. Some students even went across to the other side of the page which will definitely not be seen by the marker when the pages are chopped and scanned.

DO write legibly – the marker should not have to wrack the brains to figure out what you have written.

DO work down the page. Haphazard working makes it hard for the marker to follow your working.

DO indicate clearly if you have continued the response at the back of the booklet or reattempted a question at the back.

DO finalise your answer including paying attention to the accuracy required.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 Which of the following is equivalent to $\sqrt{3}+\sqrt{27}-\sqrt{18}$?

A. $\sqrt{3}$

B. $\sqrt{12}$

C. $4\sqrt{3}-3\sqrt{2}$

D. $10\sqrt{3}-9\sqrt{2}$

2 Which of the following graphs best represents $y=x^2+2x-3$?

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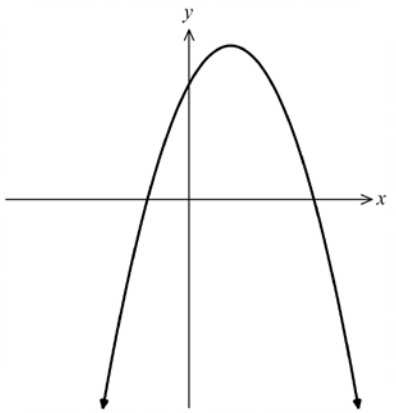
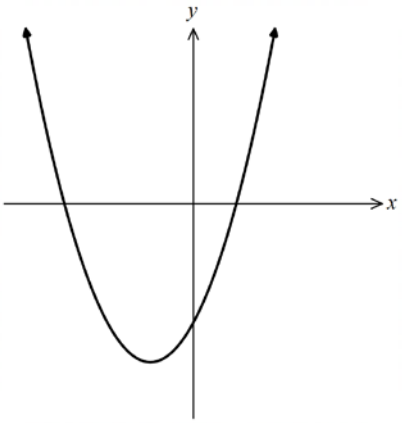
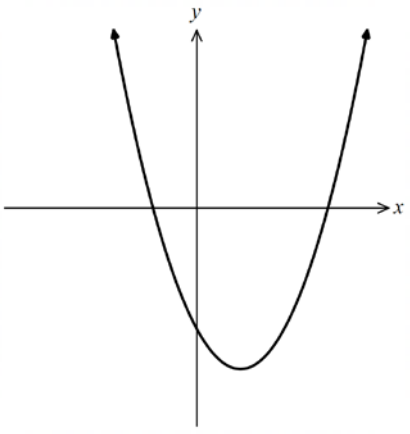


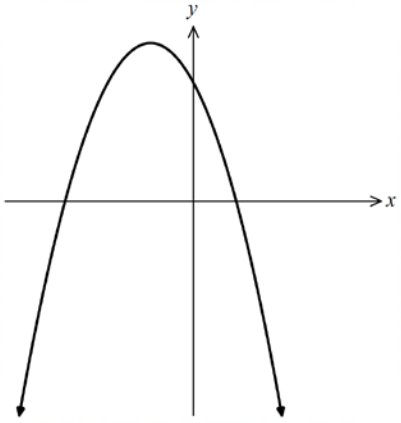
B.



C.



D.



3 Two standard unbiased dice (with faces numbered 1 to 6) are rolled.
What is the probability of rolling at least one six?

A. $\frac{1}{6}$

B. $\frac{11}{36}$

C. $\frac{25}{36}$

D. $\frac{5}{6}$

4 What is the solution to the inequation $x^2 + 4x + 3 \geq 0$?

A. $x \leq -1$ or $x \leq -3$

B. $x \leq -1$ or $x \geq -3$

C. $x \geq -1$ or $x \geq -3$

D. $x \geq -1$ or $x \leq -3$

5 Which of the following is $\frac{dy}{dx}$ if $y = 2\tan(3-2x)$?

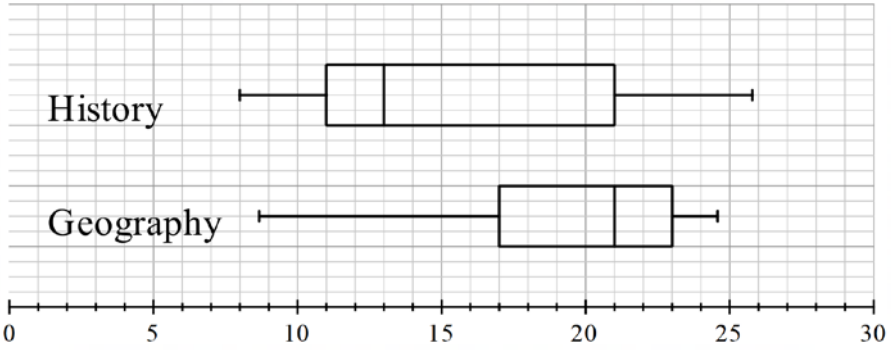
A. $\frac{dy}{dx} = 2\sec^2(3-2x)$

B. $\frac{dy}{dx} = 6\sec^2(3-2x)$

C. $\frac{dy}{dx} = -2\sec^2(3-2x)$

D. $\frac{dy}{dx} = -4\sec^2(3-2x)$

6 The box-and-whisker plots below show the results of a History and Geography test.



In History 116 students completed the test. The number of students who scored above 21 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 21

B. 42

C. 58

D. 116

7 For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > -\frac{1}{6}$

C. $x < -6$

D. $x > 6$

8 A function is defined by $y = \sin^2 \sqrt{x-2}$.
Which of the following gives the domain and range of this function?

A. Domain: $x \geq 0$ and Range: $-1 \leq y \leq 1$

B. Domain: $x \geq 2$ and Range: $-1 \leq y \leq 1$

C. Domain: $x \geq 0$ and Range: $0 \leq y \leq 1$

D. Domain: $x \geq 2$ and Range: $0 \leq y \leq 1$

9 The only stationary point on the graph $y = f(x)$ is the point (a,b).
What are the coordinates of the only stationary point on the graph $y = -f(2x)$?

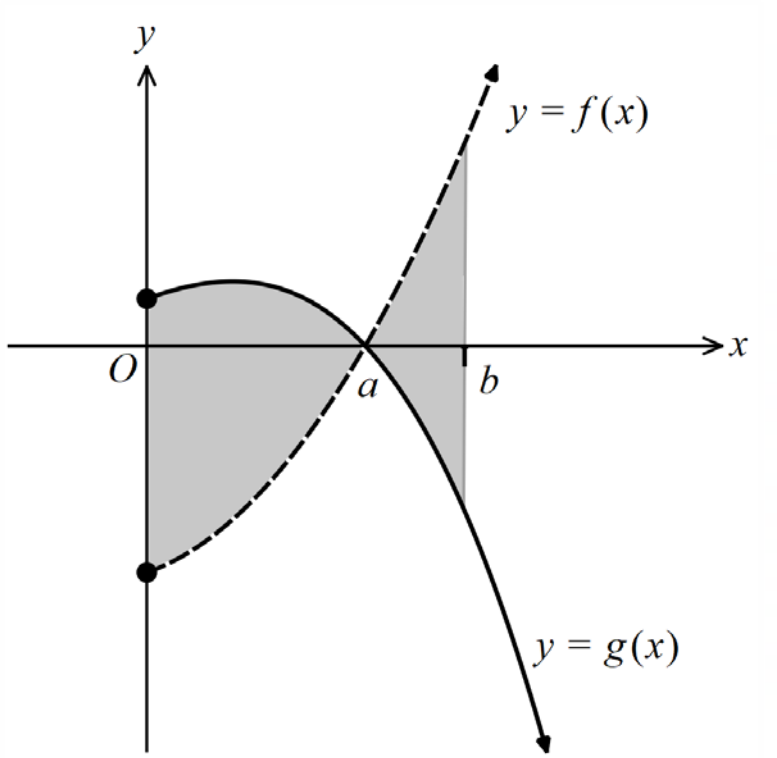
A. $\left(\frac{1}{2}a, -b\right)$

B. $(2a, -b)$

C. $\left(-\frac{1}{2}a, b\right)$

D. $(-2a, b)$

10 The functions $f(x)$ and $g(x)$ are shown in the diagram below.



Which of the following is NOT a correct expression for the shaded area?

A. $A = \int_0^b \left| f(x) - g(x) \right| dx$

B. $A = \int_0^b \left(\left| f(x) \right| - \left| g(x) \right| \right) dx$

C. $A = \int_0^b \left| f(x) \right| dx + \int_0^b \left| g(x) \right| dx$

D. $A = \int_0^a \left(g(x) - f(x) \right) dx + \int_a^b \left(f(x) - g(x) \right) dx$

Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–35
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–22 (33 marks)
Booklet 2 – Attempt Questions 23–29 (30 marks)
Booklet 3 – Attempt Questions 30–35 (27 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

100 users of a social media app were surveyed and their ages are shown in the following table.

Age	$15 \le x < 25$	$25 \le x < 35$	$35 \le x < 45$	$45 \le x < 55$	$55 \le x < 65$
Number of app users	41	30	15	8	6
Class Centre	20	30	40	50	60

Calculate the mean and standard deviation for this data, correct to two decimal places. **2**

$\bar{x} = 30.80$ $\sigma = 11.89$ to 2d.p.
Generally well done. Students are encouraged to write 30.80 for 2 decimal places. Marks not deducted for this.

Question 12 (2 marks)

Evaluate $2\int_0^1 7^{-2x}\,dx$ correct to three decimal places. **2**

$= \frac{2}{-2\ln 7}\left[7^{-2x}\right]_0^1$ $= 0.503$ to 3 d.p.
For incorrect rounding, no mark deduction this time, but students are encouraged to be careful in rounding. Half mark deducted for leaving the answer in exact form. Students are encouraged to read the question carefully, the question says 'correct to 3 decimal places'. Some students made a mistake with coefficients and ended up with $-\frac{2}{\ln(7)}7^{-2x}$. ISE marks was allowed this case. Some students tried to integrate as if it was a polynomial function, having 7^{-2x+1}. No ISE marks allowed in this case.

Question 13 (3 marks)

Find $\dfrac{d}{dx}\left(\dfrac{e^{2x}}{\sin(x)}\right)$. **3**

$$= \frac{\sin x.2e^{2x} - e^{2x}\cos x}{\sin^2 x}$$
$$= \frac{e^{2x}\left(2\sin x - \cos x\right)}{\sin^2 x}$$

Generally well done.

Question 14 (2 marks)

The fourth term of a geometric series with common ratio $r = -0.9$ is -21141.

(a) Find the first term. **1**

$T_4 = ar^3 = -21141$

$\therefore \quad a = \dfrac{-21141}{(-0.9)^3}$

$a = 29000$

Generally well done.

(b) Find the sum of the first 18 terms, correct to one decimal place. **1**

$$S_{18} = \frac{29000\left(1-(-0.9)^{18}\right)}{\left(1-(-0.9)\right)}$$
$$= 12972.2$$

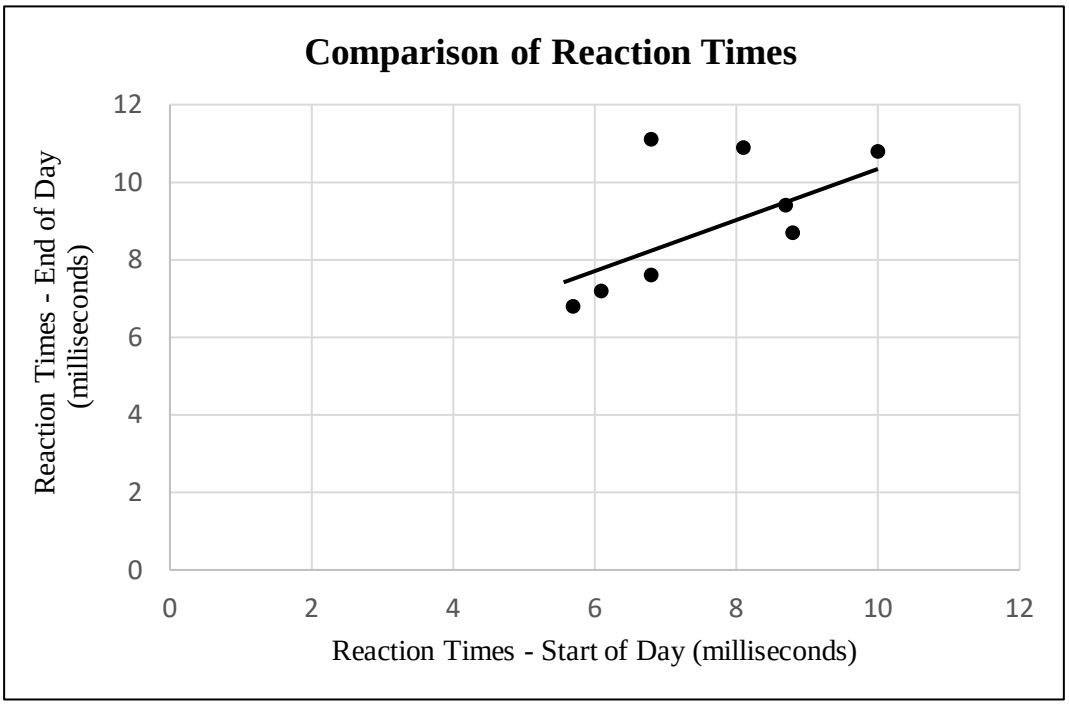
(a) ISE mark allowed for using the incorrect answer from part (a), where all the other working out is correct.

Surprisingly many students did $\left(1-(-0.9)^{18}\right) = \left(1+0.9^{18}\right)$. Half mark deducted.

Question 15 (6 marks**)**

A random sample of eight students is selected from a school database. Each student's reaction time is measured at the start and end of the school day. The reaction times in milliseconds are shown in the table and scatterplot below.

Student	A	B	C	D	E	F	G	H
Reaction time: Start of day	10.0	6.1	8.8	5.7	8.7	8.1	6.8	6.8
Reaction time: End of day	10.8	7.2	8.7	6.8	9.4	10.9	11.1	7.6



(a) Sketch a line of best fit on the graph above. **1**

Generally well done.

(b) Find the correlation coefficient and describe the association in this data. **2**

$r = 0.63$ Moderate, positive.
Generally well done. A few students described as 'moderately strong', which was not accepted.

Question 15 continues on Page 13

(c) Find the equation of the least squares regression line. Give coefficients to two decimal places. **1**

$y = 3.43 + 0.74x$ x is reaction time at the start of the day y is reaction time at the end of the day
Generally well done.

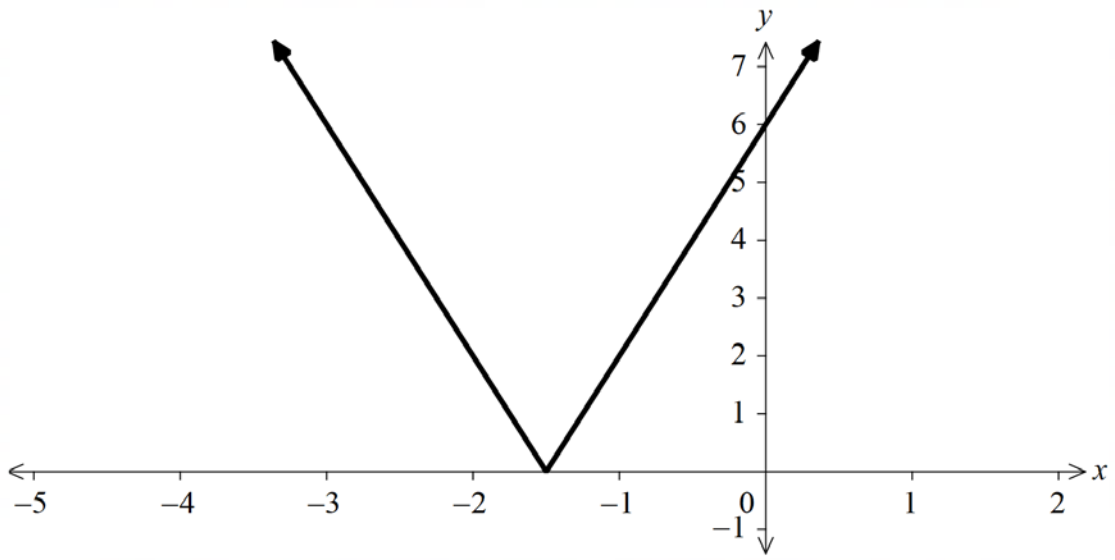
(d) Use your equation in (c) to predict the reaction time at the end of the day for a student who had a reaction time of 6.8 seconds at the start of the day.
Compare this value with the data for the relevant students from the table. **2**

When x is 6.8, $y = 8.5$. This is in between the two students measured.
ISE allowed for substituting 6.8 into the equation generated from part (c). Comparison needs to be addressed, rather than merely stating the corresponding values, to get the second mark.

End of Question 15

Question 16 (2 marks)

The graph of $f(x) = |ax + b|$ is shown below. **2**



Find the value of a and b.

$y = |ax + b|$

Sub in $x = 0, \quad y = 6$:

$6 = |b|$

$\therefore b = \pm 6$

Sub in $x = -1.5, \quad y = 0$

$0 = \left|-\frac{3}{2}a \pm 6\right|$

$0 = -1.5a + 6$ or $0 = -1.5a - 6$

$a = \frac{-6}{-1.5}$ or $a = \frac{6}{-1.5}$

$a = 4$ $\quad$ $a = -4$

Solutions are $a = 4, b = 6$ or $a = -4, b = -6$

Majority of students were able to get one set of solutions, but not many were able to find the second set of solutions. Most responses showed little or no working, using the gradient and y-intercept of the right hand branch to find $a = 4, b = 6$. Perhaps as a consequence, they overlooked the solutions relating to the second branch.

Algebraically, $|4x + 6| = |-(4x + 6)| = |-4x - 6|$. So $a = -4, b = -6$ works too.

You were awarded credit here for bald answers but in a "Find …" question you should show your working.

Question 17 (3 marks)

Solve $\dfrac{\cos^3\theta}{\sin\theta}+\sin\theta\cos\theta=1$ in the domain $0\le\theta\le 2\pi$. **3**

$\cos^3\theta+\sin^2\theta\cos\theta=\sin\theta$

$\cos\theta\left(\cos^2\theta+\sin^2\theta\right)=\sin\theta$

$\therefore \cos\theta=\sin\theta$

$\tan\theta=1$

$\theta=\dfrac{\pi}{4},\ \dfrac{5\pi}{4}$

Reasonably well done. Common errors included multiplying the equation by $\sin\theta$ to eliminate the fraction but omitting to multiply the RHS of the equation. Unfortunately, this error simplifies the subsequent working and was penalised accordingly. Writing down what you are doing (eg multiply by $\sin\theta$) can help you remember to multiply both sides of the equation.

Another error made by several students was to incorrectly simplify $\sin\theta=\cos\theta$ to $\tan\theta=0$.

Question 18 (4 marks)

The temperature T inside an oven, measured in degrees Celsius, m minutes after it is switched on, can be modelled by the equation $T = 200 - 175e^{-km}$, where k is a constant.

(a) Write down the initial temperature of the oven. **1**

When $m = 0$,

$T = 200 - 175e^{-0}$

$T = 25°$

Pretty well done. In part (a) a few students could not successfully evaluate the initial temperature as they incorrectly used $e^0 = 0$. Be careful with this, which also arises when evaluating definite integrals involving exponentials.

(b) If the oven reached a temperature of 150°C after 16 minutes and 15 seconds, find the value of k correct to three significant figures. **2**

When $m = 16.25,\ T = 150$,

$150 = 200 - 175e^{-16.25k}$

$\dfrac{-50}{-175} = e^{-16.25k}$

$-16.25k = \ln\dfrac{2}{7}$

$k = \dfrac{-\ln\left(\frac{2}{7}\right)}{16.25} = \dfrac{4}{65}\ln\left(\dfrac{7}{2}\right) \approx 0.0771$ to 3 s.f.

In part (b) most students could manipulate the resulting exponential equation correctly. Common errors tended to be in using the calculator incorrectly or in rounding correctly to three significant figures. Revisit significant figures if required.

(c) What will be the temperature in the oven after 10 minutes? **1**

When $m = 10$,

$T = 200 - 175e^{-10\times\frac{4}{65}\ln\left(\frac{7}{2}\right)}$

$= 119.05°\text{C}$

Part (c) was generally well done barring a few calculator errors. Exercise care to enter expressions correctly into the calculator. Using the ANS key or a pronumeral using the STO function on your calculator can make it easier to enter complex expressions.

Question 19 (2 marks)

A shoe store buys shoes for \$50 a pair and sells them for \$80. If their weekly rent and salary costs are \$2100 a week, how many pairs must they sell each week to break even? **2**

let x be the number of shoes sold

$R = 80x, \quad C = 50x + 2100$

for break even $R = C$

$80x = 50x + 2100$

$30x = 2100$

$x = 70$ pairs

Almost all students were successful in obtaining the correct answer and were generously awarded full credit, however there is much scope for improvement in setting out. It was not uncommon to see the following, apropos of nothing:

$80 - 50 = 30$ (a true fact, but what is this calculating?)

$\frac{2100}{30} = 70$ (again a correct numerical fact unexplained)

To improve, students should:

- Introduce a pronumeral before using it.
- Start with the definition of "breakeven" - costs equal revenues
- Answer the question

Question 20 (2 marks)

Find $\int x\sin(x^2)dx$. **2**

$= \frac{1}{2}\int 2x\sin(x^2)dx$

$= -\frac{1}{2}\cos(x^2) + C$

Very well done. Most students had no difficulty in recognising the correct primitive form. Those who missed out are advised to practise using the reference sheet to recognise a variety of derivatives and integrals.

Question 21 (2 marks)

Evaluate 10^x, given that $\frac{4^x}{2^{x+1}} = \frac{3}{5^x}$. (You are not required to find the value of x.) **2**

$(4\times5)^x = 3\times2^{x+1}$

$20^x = 3\times2^{x+1}$

$20^x = 6\times2^x$

$\frac{20^x}{2^x} = 6$

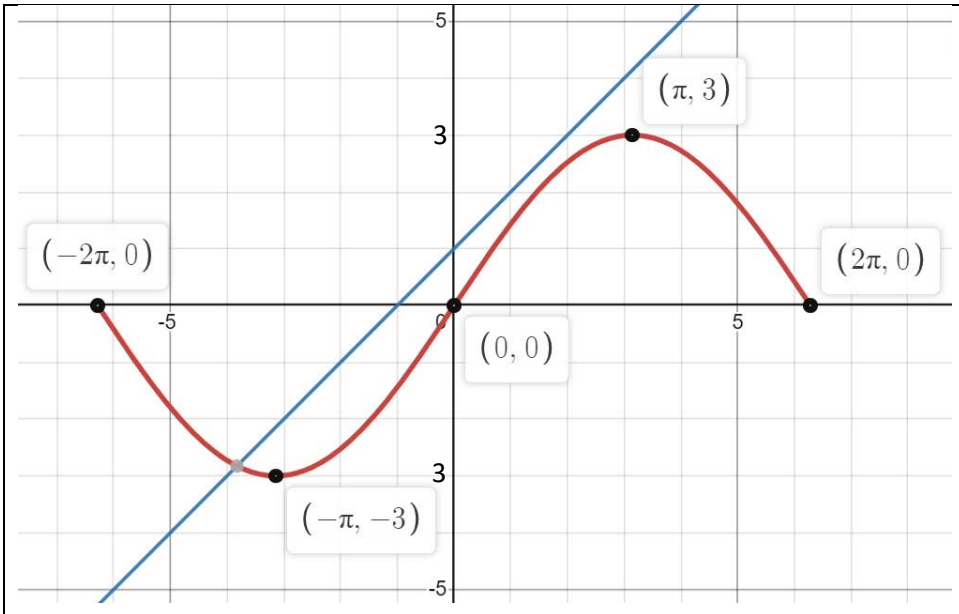
$10^x = 6$

Inconsistent performance. Those who were confident with index laws had little difficulty with this. Others need to address some fundamental misconceptions relating to indices. A common error was in combining terms with different bases for eg $2\times5^x = 10^x$ or using the distributive law incorrectly over multiplication eg $3\left(2^{x+1}\right) = 3\left(2^x.2\right) = 3.2^x.6$

With such questions try to combine terms with the same base or terms with the same index. At times the base may need to be simplified eg $4^x = \left(2\times2\right)^x = 2^x\times2^x$.

Question 22 (3 marks)

(a) Sketch the graph of $y = 3\sin\left(\frac{x}{2}\right)$, for the interval $-2\pi \le x \le 2\pi$. **2**



Part (a) was generally well done with students successfully recognising the change in amplitude and period. There is scope for improvement in the quality of the sketch. Students are advised to:

- Use a ruler to draw the axes.
- Use the ruled lines for the horizontal axes and also to help maintain scale – so many students had 3 and –3 hanging between two ruled lines.
- Practise drawing a good sine curve – some drew very rounded curves, others drew sharp mountains. Mostly this was not penalised but in some cases the shape of the curve led to incorrect conclusions in part (b).

(b) Hence or otherwise find the number of solutions to the equation $3\sin\left(\frac{x}{2}\right) = x+1$ in the domain $-2\pi \le x \le 2\pi$. **1**

The solutions to $3\sin\left(\frac{x}{2}\right) = x+1$ are the intersection points of $y = 3\sin\left(\frac{x}{2}\right)$ and $y = x+1$.

1 Solution (from graph)

Part (b) was less well done. Several students offered bald answers with no indication of how they knew how many solutions there are. The question leads with "hence or otherwise …" so students are advised to make a link to the previous part.

For instance, "The solutions to $3\sin\left(\frac{x}{2}\right) = x+1$ are the intersections between the graphs of $y = 3\sin\left(\frac{x}{2}\right)$ and $y = x+1$. From the graph in (a) it can be seen that there is only one intersection hence there is only one solution"

Drawing in the line on the previously drawn graph is a good idea – however, remember that when using graphs to solve equations, they need to be drawn to scale.

Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 23–29 (30 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 23 (4 marks)

Given that $f(x) = x^2 + 3$ and $g(x) = x + 4$:

(a) Find simplified expressions for $f(g(x))$ and $g(f(x))$. **2**

$$f(g(x)) = (x+4)^2 + 3$$
$$= x^2 + 8x + 19$$
$$g(f(x)) = (x^2 + 3) + 4$$
$$= x^2 + 7$$

Well done

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **2**

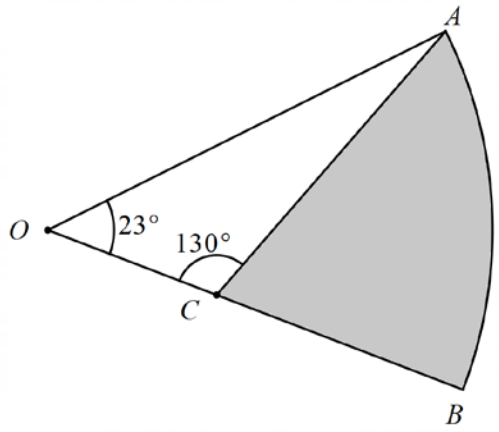
$$f(g(x)) + g(f(x)) = x^2 + 8x + 19 + x^2 + 7$$
$$= 2x^2 + 8x + 26$$
$$\Delta = 8^2 - 4 \times 2 \times 26$$
$$= -144$$
$$< 0$$

Therefore no real roots.

Reasonably well done. Students should use the discriminant. They don't need to solve the quadratic equation

Question 24 (5 marks)

The diagram shows *OAB*, the sector of a circle centred at *O*. Angle $\angle AOB$ is 23° and angle $\angle ACO$ is 130°. The line segment *AC* divides the sector and is 7 cm long.



NOT TO SCALE

(a) Find the length of *OA*, the radius of the circle. **2**

In ΔOCA $\frac{OA}{\sin 130°} = \frac{AC}{\sin 23°}$ $\therefore OA = \frac{7\sin 130°}{\sin 23°}$ $= 13.7$
Well done.

(b) Find the area of the shaded region. **3**

$A_{sector} = \frac{23°}{360°}\pi(13.7)^2$ $= 37.672\,\text{cm}^2$ $A_{\Delta} = \frac{1}{2}7\times 13.7\sin 27°$ $= 21.806\,\text{cm}^2$ $\therefore A_{shaded} = 15.903\,\text{cm}^2$ (1dp)
Some students could not see the difference between following 2 formulas: $A_{sector} = \frac{\theta}{360}\times\pi r^2$ where θ is in degrees and $A_{sector} = \frac{1}{2}\times r^2\theta$ where θ is in radians

Question 25 (2 marks)

Differentiate $f(x) = 3 - x^2$ from first principles. **2**

You may use $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\left(3 - (x+h)^2\right) - \left(3 - x^2\right)}{h} \\
&= \lim_{h \to 0} \frac{\left(3 - x^2 - 2xh - h^2\right) - \left(3 - x^2\right)}{h} \\
&= \lim_{h \to 0} \frac{-2xh - h^2}{h} \\
&= \lim_{h \to 0} \left(-2x - h\right) \\
f'(x) &= -2x
\end{aligned}
$$

Well done.

Question 26 (3 marks)

A discrete random variable X has the following probability distribution.

x	2	4	6	10
$P(X = x)$	0.3	b	0.1	c

It is known that the expected value, $E(X)$, is 4.8.

(a) Show that the values of b and c are 0.4 and 0.2 respectively. **2**

$0.3 + b + 0.1 + c = 1$

$\therefore b + c = 0.6$

$10b + 10c = 6 \qquad (1)$

$E[X] = \sum_{i=1}^{4} x_i P(X = x_i)$

$4.8 = 2 \times 0.3 + 4b + 6 \times 0.1 + 10c$

$\therefore 4b + 10c = 3.6 \qquad (2)$

$(1) - (2)$

$6b = 2.4$

$b = 0.4$

$\therefore c = 0.2$

Well done.

(b) Calculate the variance for this probability distribution. **1**

$0.3+b+0.1+c=1$

$\therefore b+c=0.6$

$10b+10c=6 \qquad (1)$

$E\left[X^2\right]=2^2\times0.3+4^2\times0.4+6^2\times0.1+10^2\times0.2$

$Var\left[X\right]=E[X^2]-\left(E[X]\right)^2$

$=31.2-4.8^2$

$=8.16$

Generally well done.

To improve, students should:

1. Work out the value of $E\left[X^2\right]=\sum x_i^2\times P\left(x_i\right)$
2. Use the formula: $Var\left(X\right)=E\left[X^2\right]-\left(E\left[X\right]\right)^2$ where $E\left(X\right)=4.8$

Question 27 (7 marks**)**

Consider the curve $y=3x^4-4x^3+2$.

(a) Find the coordinates of any stationary points on the curve and determine their nature. **3**

$y'=12x^3-12x^2$

$=12x^2(x-1)$

$y''=36x^2-24x$

for stat points $y'=0$

$\therefore x=0,1$

$y=2,1$

$y''(0)=0$

$\therefore$ horizontal inflection point at $(0,2)$

$y''(1)=12>0$

$\therefore$ minimum at $(1,1)$

In part (a) Generally well done. Students should know the difference between point of inflection and stationary point of inflection.

(0, 2) is a stationary point of inflection because both $f'\left(0\right)=0$ and $f''\left(0\right)=0$.

Or a gradient table could be used.

Most students were successful in obtaining the correct stationary points and their nature and were generously awarded full credit.

(b) Find the coordinates of any points of inflection on the curve. **2**

for inflection points $y'' = 0$

$36x^2 - 24x = 0$

$12x(3x - 2) = 0$

$\therefore x = 0, \frac{2}{3}$

Horizontal point of inflection at (0,2) shown in (a)

x	1/3	2/3	1
y"	-4	0	12
	$\cap$	$-$	$\cup$

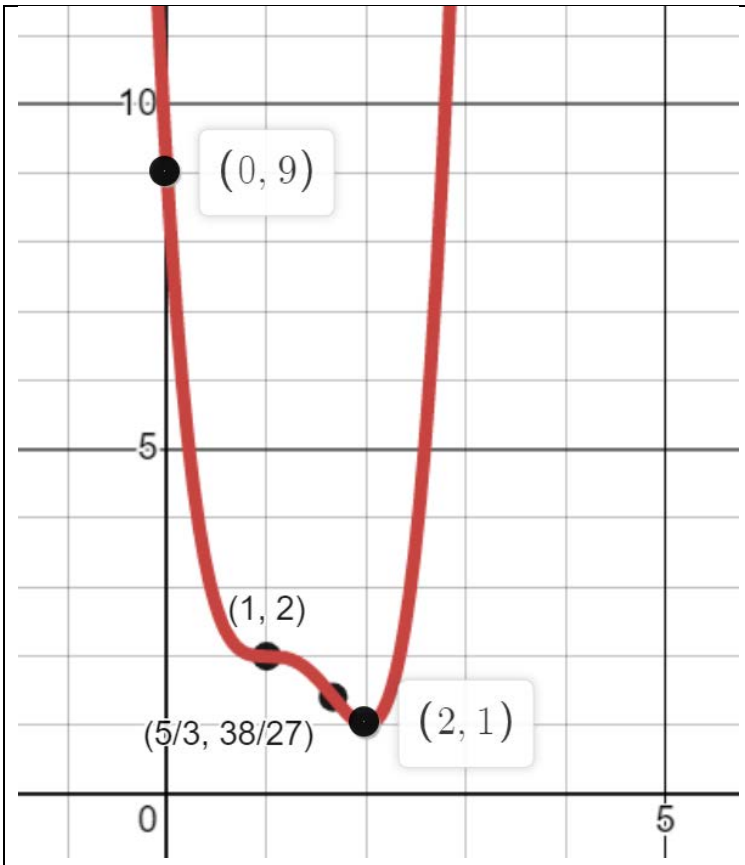
Therefore points of inflection at (0, 2) and (2/3, 38/27)

Part (b) many students did not construct a concavity table.

To improve, students should:

1. Find $f''(x)$
2. At $f''(x) = 0$, solve and find the point
3. Construct a concavity table

(c) Hence, or otherwise, sketch the graph of $y = 3(x-1)^4 - 4(x-1)^3 + 2$, showing the coordinates of the stationary points and points of inflection. **2**

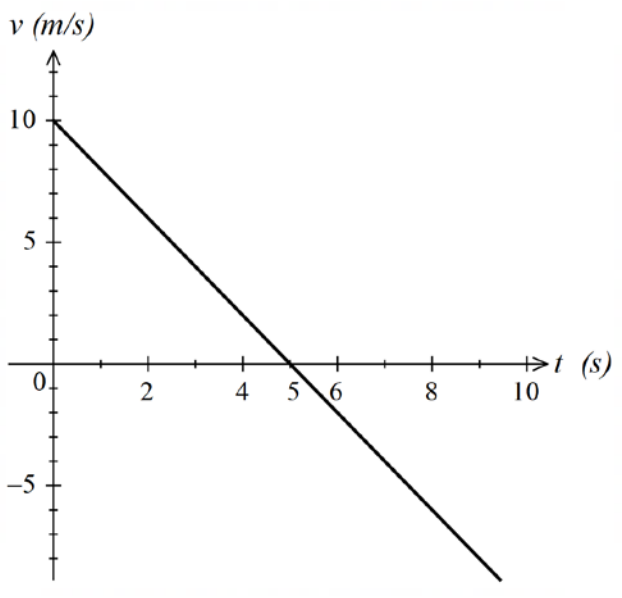


Part (c) Students should shift the curve $y = 3x^4 - 4x^3 + 2$ to the right 1 unit.

End of Question 27

Question 28 (6 marks)

The velocity-time graph of a particle moving in a straight line is shown below.



(a) Describe the motion of the particle in the first 8 seconds of the motion. **2**

The particle moved in the positive direction for the first 5 seconds, with decreasing speed. The particle came to a stop at 5 seconds, before reversing direction at increasing speed. The entire time, the particle had constant acceleration in the negative direction.
Part (a) Students should use the size of velocity to work out the direction (right or left or rest) of the motion.

(b) Find the acceleration of the particle. **1**

$a = \frac{-10}{5} = -2\,\text{ms}^{-2}$
Very well done.

Question 28 continues on Page 29

Question 28 (continued)

(c) Find the total distance travelled by the particle in the first 8 seconds. **2**

$$\begin{aligned}\text{distance} &= \frac{1}{2}\times 10\times 5+\frac{1}{2}\times 3\times 6\\ &= 25+9\\ &= 34\,\text{m}\end{aligned}$$

Many students used integrals and made too many mistakes.

To improve, students should use areas of triangles

(d) If initially the particle is two units to the left of the origin, what is the location of the particle after 8 seconds? **1**

$$\begin{aligned}\text{displacement} &= 25-9-2\\ &= 14\,\text{m}\end{aligned}$$

Generally well done.

Question 29 (3 marks**)**

Solve the equation $\tan^2\left(x-\frac{\pi}{6}\right)+\left(\sqrt{3}+1\right)\tan\left(x-\frac{\pi}{6}\right)=-\sqrt{3}$, where $0\le x\le\pi$. **3**

Give answers in exact form.

$$\tan^2\left(x-\frac{\pi}{6}\right)+\left(\sqrt{3}+1\right)\tan\left(x-\frac{\pi}{6}\right)+\sqrt{3}=0$$

$$\left(\tan\left(x-\frac{\pi}{6}\right)+\sqrt{3}\right)\left(\tan\left(x-\frac{\pi}{6}\right)+1\right)=0$$

$$\tan\left(x-\frac{\pi}{6}\right)=-\sqrt{3} \quad \text{or} \quad \tan\left(x-\frac{\pi}{6}\right)=-1$$

$$\therefore x-\frac{\pi}{6}=\frac{2\pi}{3} \quad \text{or} \quad x-\frac{\pi}{6}=\frac{3\pi}{4}$$

$$\therefore \quad x=\frac{5\pi}{6} \quad \text{or} \quad x=\frac{11\pi}{12}$$

Not done well, most students solved the equation by using the formula of quadratic equation which was too long and too hard to get the exact values.

To improve, students should:

1. Students should factorise the quadratic equations.
2. Let each factor be 0 and find their exact values
3. Reject some answers because of the given domain

Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 30–35 (27 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 30 (4 marks)

Brendan and Michelle are taking turns in an eating competition. **4**
In the first round, Brendan eats one M&M, with a volume of 0.5 cm^3, or half a millilitre, and Michelle then follows by eating two M&Ms.
In the second round, Brendan eats three M&M n Michelle eats four.
If they continue in this manner, how many M&Ms will Brendan eat in the final round before he runs out of room in his stomach, if it can hold 3 Litres?

The pattern is:
B: 1
M: 2
B: 3
M: 4 etc.
So Brendan is eating 1, 3, 5… which is an AP.

$a = 1,\ d = 2$

$T_n = 1 + 2(n-1)$

He can eat a maximum of $3000 \div 0.5 = 6000$ in total.

$$S_n = 6000 = \frac{n}{2}\left(2 \times 1 + 2(n-1)\right)$$
$$6000 = n(1 + n - 1)$$
$$6000 = n^2$$
$$n = \sqrt{6000}$$
$$n = 77.45 \qquad (n > 0)$$

Therefore game ends in 77 rounds, as Brendan can't fit in the next round.
$T_{77} = 1 + 2(77-1) = 153$
He will eat 153 M&Ms in the final round.

Done well overall. Some students didn't recognise that Brendan and Michelle were 2 separate series and that they only needed the series for Brendan, or they got confused between the 2 versions of the series (either a=0.5, d=1 OR a=1, d=2) and had some variation which combined the 2 (e.g. a=0.5, d=2). Others made small errors solving the quadratic equation or during the rounding. In this question it is important to round down because his stomach can't hold more than 3L!

Question 31 (5 marks)

A bag contains 2 white and 5 yellow tokens.
In a group of people, each person draws a token randomly from the bag, with replacement.

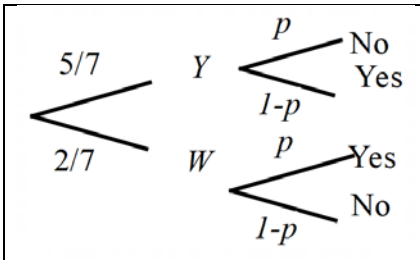
If the token is white, they answer the question "Are you a smoker?"
If a yellow token is drawn, they answer the question "Are you a non-smoker?"

They then answer either "Yes" or "No", depending on whether they smoke or not.

Let p be the probability that a randomly selected person is a smoker.

(a) Show that the probability that a randomly selected person answered "Yes" is given by: **1**

$$P(Yes) = \frac{5-3p}{7}.$$



$$P(Yes) = P(W\ Yes) + P(Y\ Yes)$$
$$= \frac{2p}{7} + \frac{5(1-p)}{7}$$
$$= \frac{5-3p}{7}$$

Generally done well. Students are recommended to use a tree diagram.

(b) It is now known that the group has 91 people and 50 of them answered "Yes".

(i) Find p. **2**

$$\frac{50}{91} = \frac{5-3p}{7}$$
$$p = \frac{5}{13}$$

Done well. Some students used 50 instead of 50/91 which resulted in p=-115. (Students are reminded to consider if their answer makes sense. ie probability must be between 0 and 1)

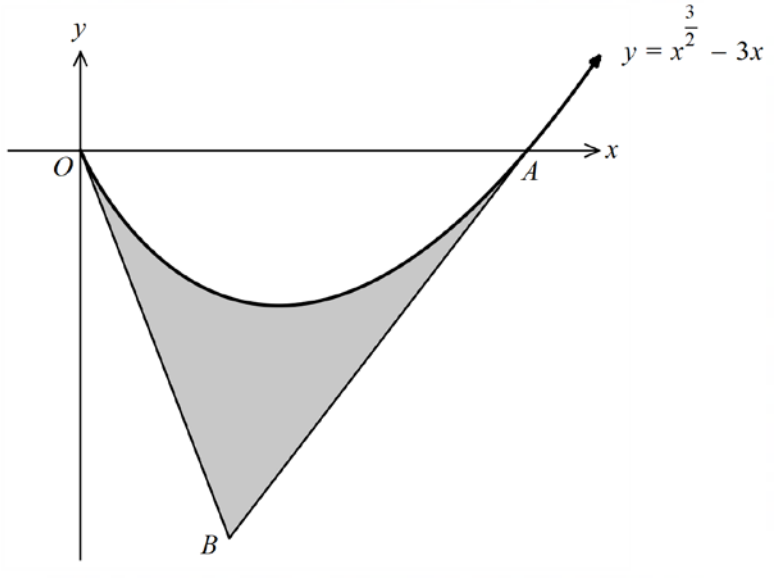
(ii) Given a person answered "No", find the probability that they are a non-smoker. **2**

$$P(\text{non-S} \mid \text{No}) = \frac{P(\text{non-S} \cap \text{No})}{P(\text{No})}$$
$$= \frac{\frac{8}{13} \times \frac{2}{7}}{\frac{41}{91}}$$
$$= \frac{16}{41}$$

Not done well. Students can recognise that they need the conditional probability formula, however, many used the formula for independence to calculate the numerator (which is not appropriate in this situation).

Question 32 (6 marks)

The diagram shows the graph of $y = x^{\frac{3}{2}} - 3x$, which meets the x-axis at the origin and at the point $A(9,0)$. Tangents drawn to the curve at O and A meet at the point B.



(a) Show the coordinates of B are $(3,-9)$. **3**

$y' = \frac{3}{2}x^{\frac{1}{2}} - 3$

OB: at $x = 0,\ y' = -3$

$y = -3x$ (1)

AB: at $x = 9,\ y' = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x - 9)$

$y = \frac{3}{2}x - \frac{27}{2}$

intersection: $-3x = \frac{3}{2}x - \frac{27}{2}$

$-6x = 3x - 27$

$27 = 9x$

$x = 3$ sub into (1)

$\therefore y = -9,$ so the tangents meet at $B(3,-9)$

Mostly done well. Some students used the point B given to find the gradient of OB and AB, then the equations, and then POI. You cannot start with point B=(-3,9) to show that point B=(-3,9).

Question 32 continues on Page 37

(b) Hence find the area of the shaded region bounded by the two lines and the curve. **3**

$$A_{triangle} = \frac{1}{2} \times 9 \times 9 = \frac{81}{2}$$

$$A_{curve} = \left| \int_0^9 x^{\frac{3}{2}} - 3x \, dx \right|$$

$$= \left| \left[\frac{2}{5} x^{\frac{5}{2}} - \frac{3}{2} x^2 \right]_0^9 \right|$$

$$= \left| \left[\frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0 \right] \right|$$

$$= 24.3 \, \text{u}^2$$

$$A_{shaded} = \frac{81}{2} - 24.3 = 16.2 \, \text{u}^2$$

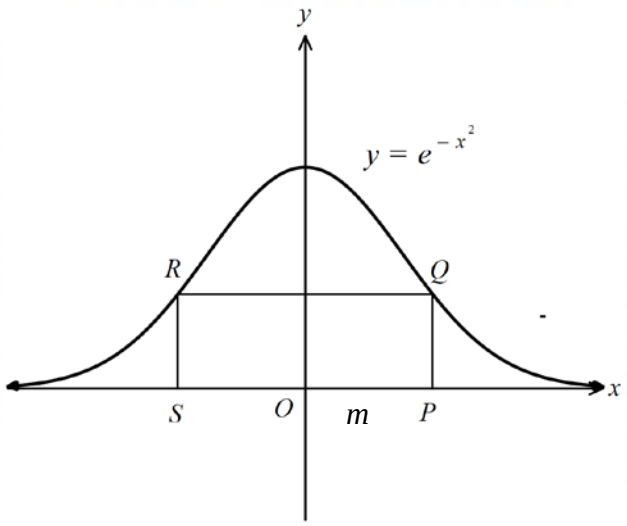
Done relatively well. There were a variety of ways students chose to approach this question. A significant portion did not recognise that to find the area of triangle AOB you could just use 1/2bh, and spent time integrating instead. Additionally, just because the question is asking for area, does not mean you randomly put in absolute values anywhere. Students need to think carefully about where they insert absolute values so it is appropriate.

End of Question 32

Question 33 (4 marks) |

The diagram shows a rectangle *PQRS*, where *Q* and *R* are points on the curve $y = e^{-x^2}$ **4**

and *P* and *S* are on the *x*-axis. *O* is the origin and $OP = OS$.



Let the length $OP = m$. Find the value of *m* that maximises the area of *PQRS*.

$y = e^{-x^2}$ is even

$A = 2 \times OP \times PQ$

$A = 2me^{-m^2}$

max area when $\frac{dA}{dm} = 0$

$$\frac{dA}{dm} = 2e^{-m^2} + 2me^{-m^2}(-2m) = 0$$

$$2e^{-m^2}\left(1 - 2m^2\right) = 0$$

$$\therefore m^2 = \frac{1}{2}$$

$$m = \frac{1}{\sqrt{2}},\ m > 0$$

m	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	1
$\frac{dA}{dm}$	0.78	0	-0.735
	/	-	\

$\therefore m = \frac{1}{\sqrt{2}}$ is max

Done well. Some students struggled with setting up the formula for area.

Question 34 (2 marks)

Given that $-\frac{\pi}{2} < x < \frac{\pi}{2}$, simplify the expression below as a single trigonometric ratio, **2**

$$\sqrt{1+\sin^2 x+\sin^4 x+\sin^6 x+\ldots}$$

consider $1+\sin^2 x+\sin^4 x+\sin^6 x+\ldots$

This is a GP with $a=1,\ r=\sin^2 x$

Since $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ r < |1|$

let $S_\infty = 1+\sin^2 x+\sin^4 x+\sin^6 x+\ldots$

$$\therefore S_\infty = \frac{1}{1-\sin^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \sqrt{S_\infty} = \sqrt{\sec^2 x} = \sec x \qquad \left(\text{since } -\frac{\pi}{2} < x < \frac{\pi}{2},\ \sec x > 0\right)$$

Students who recognised it was GP generally also recognised it was a limiting sum. However, some students did not fully simplify the expression or the simplified incorrectly. Many students thought the square root of $\sec^2 x$ was $\pm \sec x$ when it technically is $|\sec x|$. In this case since

$-\frac{\pi}{2} < x < \frac{\pi}{2},\ \sec x > 0,\ \sqrt{\sec^2 x} = +\sec x$

Question 35 (6 marks)

An engineer models the rates of change of the amount of oil produced (in hundred barrels per day) by oil companies *A* and *B* respectively by

$$f'(t) = 6\ln(t) \quad \text{and} \quad g'(t) = \frac{60t}{1+5t} \ ,$$

where *t* is the number of days after some initial time $(2 \le t \le 14)$.

(a) Using the trapezoidal rule with 3 sub-intervals, estimate the total amount of oil produced by oil company *A* from $t = 2$ to $t = 14$. **2**

$$\int_2^{14} f'(t) \approx \frac{14-2}{2\times 3}\left[6\ln(2)+6\ln(14)+2\left(6\ln(6)+6\ln(10)\right)\right]$$
$$\approx 138.25$$

Not done well. Students did not recognise that 3 subintervals is equivalent to 4 function values and hence substituted the wrong number of subintervals/ value for n into the formula. Other students incorrectly thought they had to integrate $6\ln x$ first (which is NOT 6/x) and then substituted into this to find their function values instead.

(b) It can be shown that $\frac{5t}{1+5t} = 1 - \frac{1}{1+5t}$. DO NOT prove this. **1**

Use this result to find $\int \frac{60t}{1+5t}\,dt$.

$$\int \frac{60t}{1+5t}\,dt = 12\int \frac{5t}{1+5t}\,dt$$
$$= 12\int\left(1-\frac{1}{1+5t}\right)dt$$
$$= 12\left[t-\frac{1}{5}\ln\left(1+5t\right)+C\right]$$
$$= 12t-\frac{12}{5}ln\left(1+5t\right)+C_1$$

Not done well. Students could recognise how to use the given result but then couldn't integrate or integrated incorrectly.

Question 35 continues on Page 41

(c) The engineer claims that the total amount of oil produced by oil company *A* from $t = 2$ to $t = 14$, is less than that of oil company *B*. Do you agree? Justify your answer. **3**

$$\int_2^{14} \frac{60t}{1+5t}\,dt = 12\left[t - \frac{1}{5}\ln(1+5t)\right]_2^{14}$$
$$= 12\left[14 - \frac{1}{5}\ln(1+5\times 14) - \left(2 - \frac{1}{5}\ln(1+5\times 2)\right)\right]$$
$$= 12\left[12 - \frac{1}{5}\ln\left(\frac{71}{11}\right)\right]$$
$$\approx 139.52$$

Therefore company *B* will produce approximately 139.52 hundred barrels.
From part (i), company *A* produced approximately 138.25 hundred barrels.

So from these estimates, Company *B* is producing more. However, the estimate for Company *A* was obtained using the trapezoidal rule. As the natural log function is concave down, this will be an underestimate and the values are very close. So it is not possible to say from these values which company produces more oil over that period.

Students who made mistakes in trapezoidal rule for part (a) generally made the same mistake if they used trapezoidal rule for this part too (even though they should have used their integral from part b to find the exact value instead). Students got 1 mark for a in they found the amount of oil produced by company B and compared this value directly to the value they found for company A in part (a). To get the other marks, students needed to recognise that the value in part (a) was an underestimate and hence conclude that it is not possible to tell which company made more oil.

End of paper